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A SOLUTION TO THE PROBLEM OF ADJUSTING THE COUNTERBALANCE OF A SHIPBOARD THEODOLITE

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The usual shipboard theodolite is essentially a sextant mounted on gimbals and equipped with a horizontal circle. In order to keep the sextant upright in the gimbal mounting, a shaft with a heavy weight or counterbalance on it is rigidly attached to the outer spindle of the horizontal circle assembly. On some types of instruments, such as that shown in figure 1, this weight or counterbalance may be adjusted vertically. In the use of such an instrument, then, the problem naturally arises as to what is the optimum counterbalance adjustment for a given set of conditions aboard ship.

As far as can be learned, the only reference to any attempt to solve this problem is that contained in an article written by F. Eredia on the "Exploration of the Atmosphere by Means of Pilot Balloons on Board Merchant Vessels" which was published in volume IV the "Annali dell'Ufficio Presagi" appearing in 1932. A translation[1] of his remarks regarding the problem of offsetting the effects of the motion of the ship is as follows:

> This problem has been solved by supplying a Cardan suspension (gimbal mounting) with a pendulum rigidly attached to it (the theodolite)—giving the pendulum sufficient mass since the center of gravity should be rather low. The length of the pendulum may be varied continuously in such a manner as to modify the period of oscillation so that it avoids resonance with the period of oscillation of the ship. Moreover, three springs notably reduce the oscillation of the pendulum itself.
>
> Experience has shown that, having once regulated the period of the pendulum by means of trial in the best possible manner, pilot balloon soundings can be taken with the same ease as on land.

Since conditions, as far as the motion of a given ship is concerned, vary greatly from time to time—depending upon the type of sea encountered and the ship's heading into it among other things—it is believed that it will be of considerable advantage to be able to compute the optimum position of the counterbalance for a given set of conditions aboard ship. To this end, the following investigations have been pursued:

To begin with, the problem as to the proper counterbalance adjustment for a free suspension (i. e., without springs) of the theodolite was considered and an attempt was made to apply the classical equation for forced vibrations with viscous damping to it. To do this the following assumptions and limitations were imposed initially:

1. The motion of the theodolite was considered to be confined to a single plane—this plane being that perpendicular to the axis of the most pronounced angular motion of the ship—its roll.
2. It was assumed that the theodolite had been so oriented that the axis about which the instrument oscillates within the gimbal ring was parallel to the ship's axis of roll.
3. It was assumed that it would be possible, finally, to so adjust the counterbalance that its amplitude of oscillation with respect to the vertical would be less than 3°.
4. Damping proportional to the first power of the angular speed with respect to the vertical (viscous damping) was assumed. The torques due to other types of damping were assumed to be negligible.
5. In accordance with the theory of the seismograph, the inertia effect on the part of the instrument on gimbals was assumed to be concentrated at its center of gravity.
6. The error introduced by considering the direction of the acceleration of the point of suspension due to the roll of the ship to be at all times normal to the counterbalance shaft was assumed to be negligible.
7. The effect of any torques which might arise when the theodolite moves through the air with the rolling of the ship or which might arise due to the blowing of the wind is assumed to be negligible.

Under these assumptions, then, the equation of motion is:

$$\frac{d^2\theta}{dt^2}+\frac{\kappa d\theta}{dt}+\frac{g\theta}{\lambda}=\frac{-Mh}{K}\frac{d^2y}{dt^2} \tag{1}$$

where

θ is the angle which the theodolite makes with the vertical,
κ is the factor by which the angular speed must be multiplied in order to obtain the angular acceleration due to damping,
t is the time,
g is the acceleration of gravity,
M is the mass of the part of the instrument on gimbals,
h is the distance of the point of support from the center of gravity of the part of the instrument on gimbals,
K is the moment of inertia of the part of the theodolite on gimbals,
$\lambda=\frac{K}{Mh}$ is the length of the equivalent simple pendulum,

and

y is the displacement (along the arc) of the point of support from its equilibrium position.

If, now, Y being the maximum displacement along the arc of the point of support from its equilibrium position and p being 2π divided by the period of roll of the ship, the equation

$$y=Y\cos pt \tag{2}$$

is assumed to hold, then

$$\frac{d^2y}{dt^2}=-Yp^2\cos pt \tag{3}$$

Letting Φ_0 represent the amplitude of roll of the ship and H the distance of the point of support of the instru-

[1] Furnished by Carl Russo of the Aerological Division.

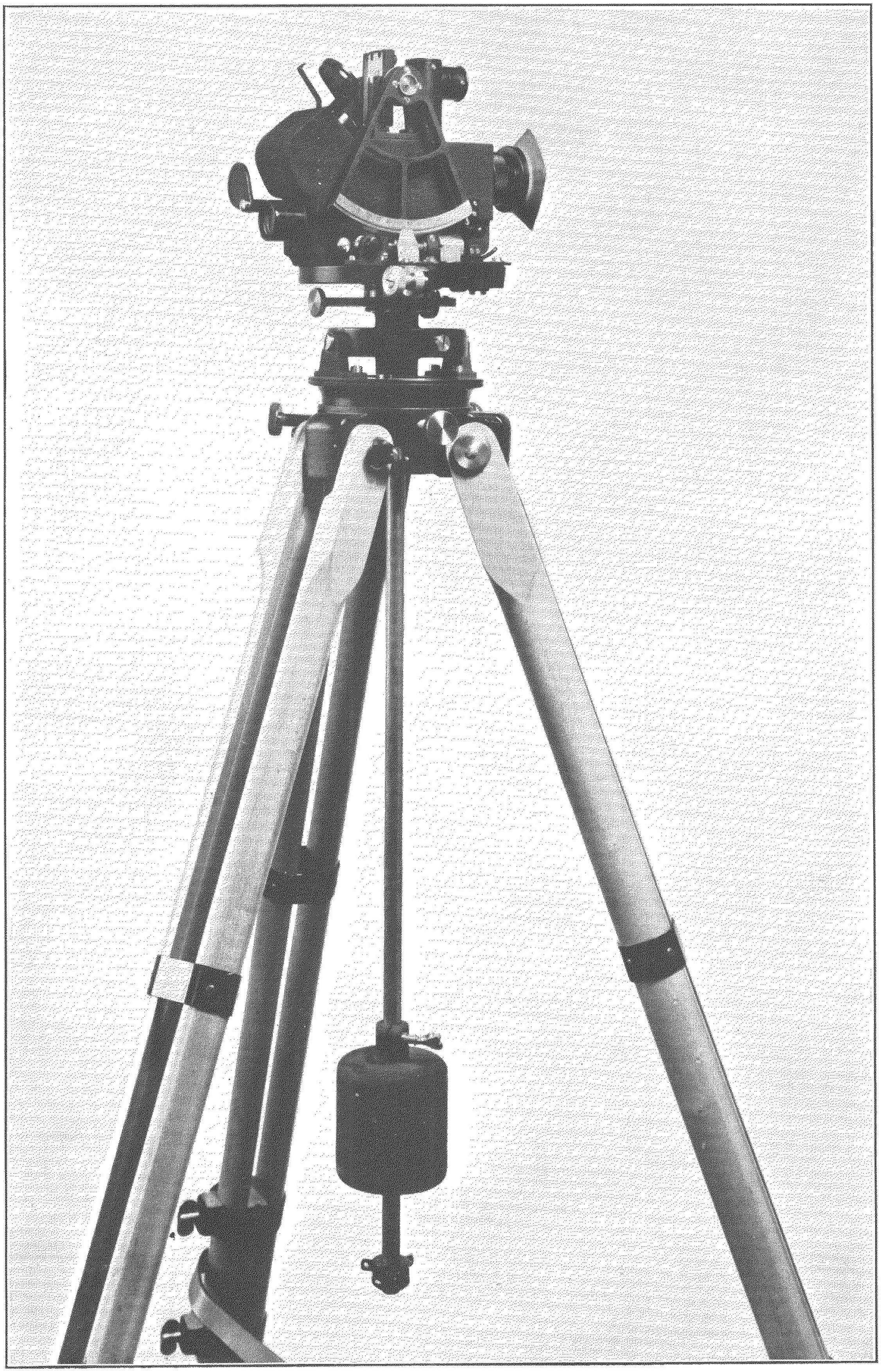

Figure 1.

ment from the axis of roll of the ship, equation (3) becomes

$$\frac{d^2y}{dt^2}=-\Phi_0Hp^2\cos pt \tag{3a}$$

Substituting in equation (1), the required differentia equation becomes

$$\frac{d^2\theta}{dt^2}+\kappa\frac{d\theta}{dt}+\frac{g}{\lambda}\theta=\frac{\Phi_0Hp^2\cos pt}{\lambda} \tag{4}$$

The expression for the amplitude of vibration obtained from this equation is found to contain, among others, the constant κ. In order to evaluate this constant, the logarithmic decrement of the theodolite was measured. It was found, however, that instead of having a constant logarithmic decrement, as is demanded by the assumption of viscous damping, the decrements in the amplitudes of free swing were approximately constant. This means that instead of the angular acceleration due to friction being represented by the expression $\kappa\frac{d\theta}{dt}$ it must be represented by a constant. This is evident from the following considerations:

If θ_1 represents the amplitude of any swing of the instrument other than any one of the final three swings before the instrument comes to rest, and θ_2 and θ_3 be the succeeding amplitudes of swing, the angular acceleration is:

$$\frac{\frac{4}{T}\left(\theta_1-\frac{\theta_1-\theta_2}{4}\right)-\frac{4}{T}\left(\theta_2-\frac{\theta_2-\theta_3}{4}\right)}{T}$$

where T represents the natural period of swing of the instrument. Letting $\Delta\theta$ represent the constant decrement of the amplitude of swing, the expression $\frac{4\Delta\theta}{T^2}$ is finally obtained for the frictional angular acceleration. Since T may, for the purposes of this investigation, be regarded as a constant, it follows then that the resulting angular acceleration is constant.

Letting F represent the value of this constant, then, the equation of motion of the theodolite assumes the form:

$$\frac{d^2\theta}{dt^2}+\frac{g}{\lambda}\theta\pm F=\frac{\Phi_0Hp^2}{\lambda}\cos pt, \tag{5}$$

the plus sign before F being used for one direction of swing and the minus sign being used for a swing in the opposite direction. A solution of this equation has been published by Den Hartog in Vol. 53 of the *Transactions of the American Society of Mechanical Engineers* (Section APM, page 107). Den Hartog, however, substitutes the expression $\cos(pt+\phi)$ for $\cos pt$ so that the equation to be solved finally becomes

$$\frac{d^2\theta}{dt^2}+\frac{g}{\lambda}\theta\pm F=\frac{\Phi_0Hp^2}{\lambda}\cos(pt+\phi). \tag{5a}$$

This change was made "for the purpose of subsequently writing the boundary conditions in a simple form." Selecting the negative sign before F the solution of this equation, which corresponds to Den Hartog's (6), is

$$\begin{aligned}\theta=\theta_0\cos p_nt&+\frac{F}{p_n^2}(1-\cos p_nt)\\&+\frac{\Phi_0Hp^2}{\lambda(p_n^2-p^2)}\Big[\cos\phi(\cos pt-\cos p_nt)\\&+\sin\phi\left(\frac{p}{p_n}\sin p_nt-\sin pt\right)\Big]\end{aligned} \tag{6}$$

where $p_n=\sqrt{\frac{g}{\lambda}}$. The values of $\cos\phi$ and $\sin\phi$ may be obtained as is shown by Den Hartog (with the exception that a positive instead of a negative sign should be used for the given values of $\sin\phi$) and there results, then, as the equation corresponding to Den Hartog's equation (9)

$$\theta_0^2=\frac{\Phi_0^2H^2p^4}{(g-\lambda p^2)^2}-\frac{L^2}{MhKgp^2}\frac{\sin^2\frac{p_n}{p}\pi}{\left(1+\cos\frac{p_n}{p}\pi\right)^2} \tag{7}$$

where $L=K\times F=$the torque due to friction.

When the function of the form $\frac{\sin u}{1+\cos u}$ contained in the last term on the right-hand side of this equation is examined, it is found to become indeterminate whenever u assumes any of the odd integral multiples of π as a value. An application of L'Hospital's rule for evaluating indeterminate forms to this function shows that it becomes infinite at these points. Since the coefficient of the square of this function does not vanish for finite distances of the counterbalance from the point of suspension and the first term on the right-hand side becomes infinite only when λp^2 approaches g as a limit (which, assuming that the period of roll varies between 6 sec. and 20 sec., can happen only if λ takes on a value lying between 10 and 10,000 meters), it might appear that this equation would give negative values of θ_0^2 at the points in question. As Den Hartog has pointed out, however, a limiting condition as to the application of (7) is furnished when it is considered that the negative sign before F was chosen in obtaining the desired solution. This, of course, means that $\frac{d\theta}{dt}$ must be considered negative for the interval for which (7) applies. When equation (6) is differentiated with respect to the time and this fact is utilized, it is found that the relation

$$\theta_0>\frac{F}{pp_n}\,\frac{\sin p_nt+\dfrac{\sin\frac{p_n}{p}\pi}{1+\cos\frac{p_n}{p}\pi}(\cos pt-\cos p_nt)}{\sin pt} \tag{8}$$

must hold for all values of t in the interval $0<t<\frac{\pi}{p}$ (assuming that $t=0$ at the beginning of the period for which the negative sign of F applies). If it can be shown now that the expression on the right-hand side of (8) is positive for any one value of t in the given interval, θ_0 must always be greater than such a value in order for (7) to apply and hence will always be positive. The existence of the value required may be shown by applying L'Hospital's Rule to the function for the case in which $t=0$ (which may be done since the function becomes indeterminate for this value of t). Upon doing this it is found that the limit $\frac{F}{pp_n}$ is approached by the function as t approaches zero from above. This, then, means that θ_0 can only be less than $\frac{F}{pp_n}$ by an infinitely small amount and the required value of the function is thus found to exist in the neighborhood of $t=0$.

If it is now assumed that equation (7) holds for all positions of the counterbalance except those eliminated by (8), it is evident that the slope of the curve so obtained will be positive on one side of the region for which (7) does not hold and negative on the other side of such a region. This indicates that a minimum value of θ_0 lies within these regions and that the position of the counterbalance for which a minimum occurs is approximately that for which the last term of equation (7) becomes infinite. On the basis of this conclusion and with the use of the data for the theodolite previously mentioned the curves shown in figure 2 have been drawn to indicate the manner in which the counterbalance should be adjusted. It is evident from this figure that if the period of roll of the ship varies more than a second from the mean period for which the counterbalance has presumably been adjusted, the position of the counterbalance will then be in the neighborhood of that for a maximum instead of a minimum. That such a variation is of an extremely frequent occurrence is shown by an inspection of the record of the roll period of the U. S. S. *Pensacola* while she was underway in a calm sea in which a moderate swell existed. The mean period of roll for 155 observations was found to be 11.5 seconds and the average deviation from this mean was found to be 1.2 seconds. Hence, even with very nearly ideal conditions as far as the sea was concerned and with a 10,000-ton ship, the variation in the period of roll was found to be quite sufficient to give just about as many maximum amplitudes of oscillation as minimum amplitudes. It is then evident that for a free suspension of the theodolite, the position of the counterbalance is practically immaterial.

If, however, the device of connecting the counterbalance shaft to the tripod legs by light springs is employed (which device was mentioned in the previously quoted article by Signor Eredia and has also been in use by the United States Navy since 1928, or before) and an absence of pitching together with a fixity of the axis of roll with respect to the ship is assumed, verticality of the theodolite at the extreme of a given roll can be attained if springs of the proper stiffness are used and the counterbalance is properly adjusted. The stiffness of the springs to be used and the way in which the counterbalance adjustment should be made is derived from the following considerations.

The center of mass will still move toward the lower side of the ship in its rolling as before. A necessary condition that the center of mass of the theodolite should be at an extreme in this motion is that there should be equilibrium between the torques exerted by all the forces involved—i. e., between the torques exerted by the forces due to the spring, friction, gravity, and the inertia of the theodolite.

Assuming (1) that one leg of the tripod lies in a plane perpendicular to the roll axes, (2) that this leg is on the opposite side of the theodolite from the lower side of the ship in the roll considered, and (3) that the springs are taut but not extended when the deck is level and the counterbalance shaft is vertical; the torque exerted by the spring under tension is given by the expression

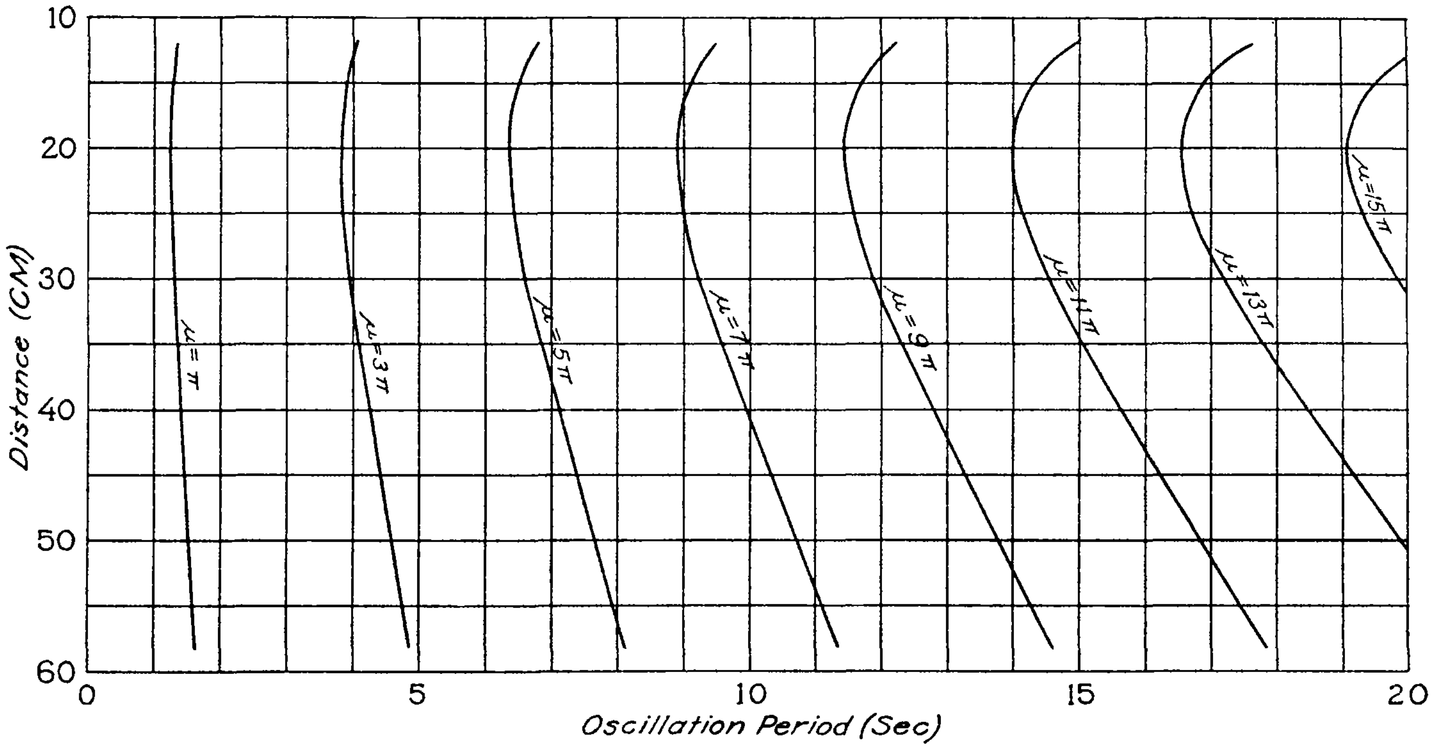

FIGURE 2.

$$kqw\,\frac{(z-r)}{z}\sin\,(\gamma+\Phi_0+\theta)$$

where, referring to figure 3:

$r=AC=$ the distance between the points of attachment of the spring to the counterbalance shaft and to the tripod leg respectively when the spring is unextended.

$q=CO=$ the distance of the point of suspension of the theodolite from the point of attachment of the spring to the tripod leg.

$w=AO=$ the distance of the point of suspension of the theodolite from the point of attachment of the spring to the counterbalance shaft.

$\gamma=\angle AOC=$ the angle formed at the point of suspension by straight lines through the points of attachment of the spring when the deck and tripod head are level and when the counterbalance shaft is vertical.

$z=$ the distance between the points of attachment of the spring at the instant considered.

$k=$ the stiffness of the spring.

For a roll in the opposite direction, the torque exerted by the two springs combined is given by the expression

$$2ksw\,\frac{(v-r)}{v}\sin\,(\alpha+\Phi_0+\theta)$$

in which, referring to figures (3) and (4),

$s=DO=$ the slant height of the pyramid formed by the point of suspension of the theodolite and the three points of attachment of the springs to the tripod legs.

$\alpha=\angle BOD=$ the angle formed by drawing lines from the point of suspension through the extremities of the altitude drawn through the mid-points of the parallel sides of the trapezoid formed by the four points of attachment of the springs concerned,

and v is given by the equation

$$v^2=a^2-2a\ \sqrt{q^2-s^2}+q^2+w^2-2ws\cos\ (\alpha+\Phi_0+\theta)$$

in which $a=A'B=A''B$ is one-half the distance between the two points at which the springs are attached to the counterbalance shaft.

The torque exerted by friction is, as has been mentioned previously, $L=K\times F=$const., K being the moment of inertia of the theodolite and F being the angular acceleration due to friction.

The torque due to the inertia of the theodolite is:

$$Mh\Phi_0Hp^2\cos\ (\Psi+\Phi_0+\theta)$$

where

$\Psi=$ the angle whose tangent is given by the ratio of the distance of the point of suspension from the medial plane of the ship (the plane which divides the ship longitudinally into two symmetrical parts) to the height of the point of suspension above the water

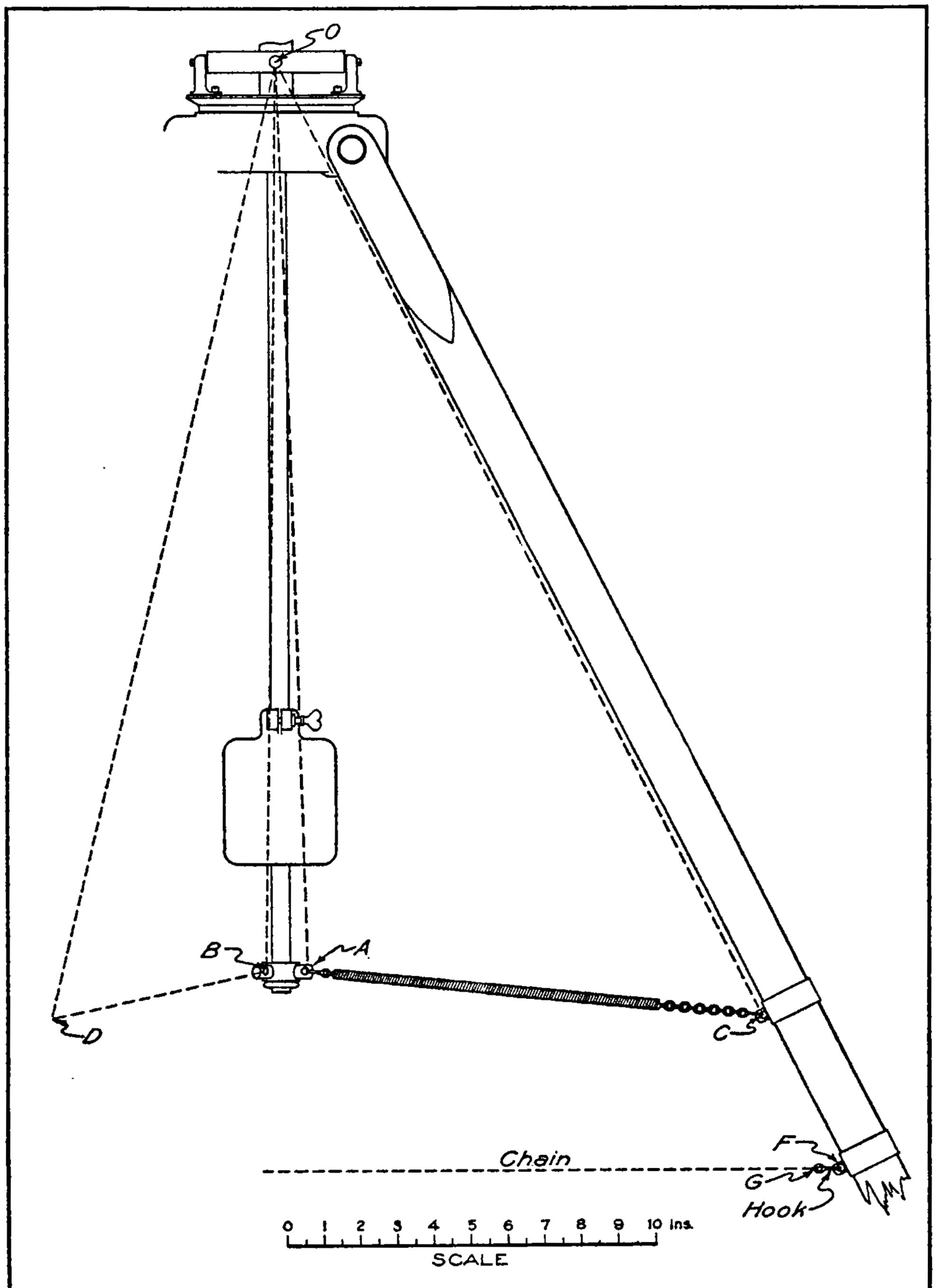

FIGURE 3.

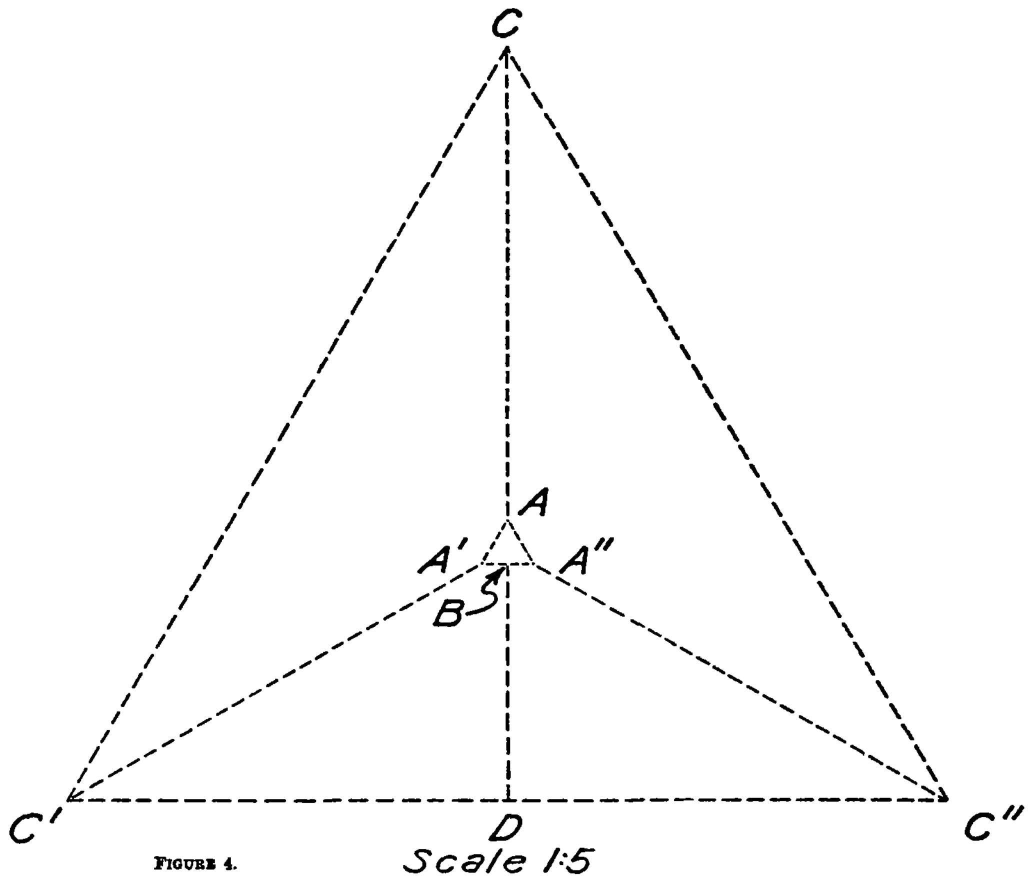

Figure 4.

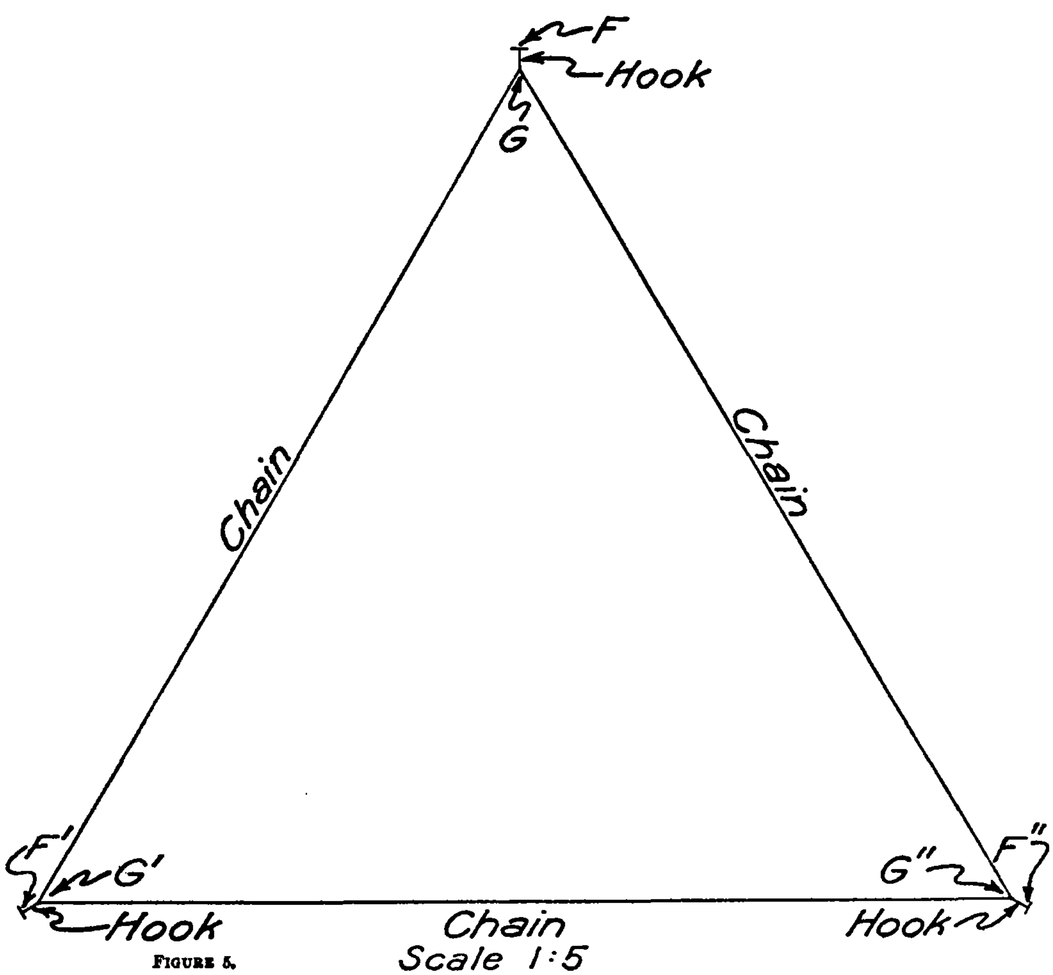

Figure 5.

plane of the ship when the ship is in its equilibrium position.

Now it is evidently possible to adjust the counterbalance so that Mh has such a value that when it is multiplied by $H\Phi_0 p^2 \cos \Phi_0$ it is equal to the torque exerted by friction plus the torque which the spring exerts for a vertical position of the theodolite when the ship is in an extreme position in its rolling. This adjustment having been made, the only value of θ which will satisfy the necessary condition set forth above for having the center of mass at an extreme position in its motion toward the lower side of the ship in its rolling is the value zero—i. e., assuming that $\Psi=0$. For, considering θ to increase as the center of mass of the theodolite (lying below the point of suspension) moves towards the lower side of the ship in its rolling, a positive value of θ at the extreme of a roll will mean that the inertial torque—acting in a positive direction—has decreased as compared with its value when $\theta=0$, while, on the other hand, the spring torque—acting negatively—has increased as compared with its value when $\theta=0$. These two changes will then produce a resultant negative torque which when augmented by the negative torque due to gravity becomes even greater. Similarly, the changes in the inertial and spring torques caused by assuming a negative value of θ, will produce a positive torque and, when the positive torque due to gravity under these circumstances is added in, the resultant torque has an even greater positive value. It then follows that the only value of θ which will satisfy the required condition is, as has been said, that of zero. Therefore, assuming that the lower side of the ship in its rolling is on the opposite side of the point of suspension from the tripod leg which lies in the plane athwart ship, the torque equilibrium equation is:

$$Mh_0\Phi_0 Hp^2 \cos \Phi_0 = L + kqw\frac{(z_0-r)}{z_0}\sin(\gamma+\Phi_0) \qquad (9)$$

where

h_0= the distance of the point of suspension from the center of mass of the theodolite corresponding to the required counterbalance adjustment.

and

z_0= the distance between the points of attachment of the spring when the theodolite is vertical at the extreme of the roll.

Now $Mh_0 = M_2x_0 + M_1x_1$

where

M_2= the mass of the counterbalance,

x_0= the distance of the point of suspension from the center of mass of the counterbalance when the required adjustment is made,

M_1= the mass of the part of the theodolite on gimbals less the mass of the counterbalance,

and

x_1= the distance of the center of mass of M_1 from the point of suspension.

Therefore,

$$x_0 = \frac{kqw}{M_2\Phi_0 Hp^2 \cos \Phi_0}\cdot\frac{z_0-r}{z_0}\sin(\gamma+\Phi_0) + \frac{L}{M_2\Phi_0 Hp^2\cos\Phi_0} - \frac{M_1x_1}{M_2} \qquad (10)$$

Similarly, for a roll in the opposite direction

$$x'_0 = \frac{2k'sw}{M_2\Phi_0 Hp^2 \cos \Phi_0}\cdot\frac{v_0-r}{v_0}\sin(\alpha+\Phi_0) + \frac{L}{M_2\Phi_0 Hp^2\cos\Phi_0} - \frac{M_1x_1}{M_2}, \qquad (11)$$

x'_0 k' and v_0 being the quantities corresponding to x_0, k, and z_0, respectively, for the direction of roll first considered.

In deriving the quantity H which appears in both (10) and (11), it is believed that it will be found sufficiently accurate to assume that the mean position of the axes of roll lies at the intersection of the medial plane of the ship and the ship's water plane. If the point of suspension lies in the medial plane of the ship, therefore, the quantity H will merely be the height of this point above the water plane of the ship when the ship is in an equilibrium position. If, however, the point of suspension does not lie the ship's medial plane, H must be taken as

$$\frac{\text{Height of suspension point}}{\text{Cos } \Psi}$$

Also, the following facts are to be borne in mind in applying these equations:

(1) Since in the determination of the constant L, the gimbal bearings were perfectly dry, all traces of anything that tends to act as a lubricant should be removed from them. This should be done, if possible, by washing them in a solvent for such lubricants.

(2) The points at which the springs are attached to the counterbalance shaft are assumed to be fixed with respect to the shaft. This means that in the case of the instrument shown in figure 1, the movable ring into which the springs are hooked must be fixed so that the ring cannot slide along the counterbalance shaft. One of the best ways to do this would seem to be that of putting a set screw in the ring in such a manner that when it is tightened all sliding on the shaft is prevented.

To facilitate the application of equations (10) and (11), all necessary measurements have been made on an instrument of the type "Aero 1928" manufactured for the U. S. Navy. Since the instrument is equipped with an extension type tripod a fixed spread of the legs was arbitrarily adopted—it being assumed that the device of stretching a given length of chain between the tripod legs would be used to give the chosen amount of spread whenever the theodolite is set up. The spread adopted was a little less than 25°. Assuming that the three lengths of chain are connected by the closed ends of three small hooks which are furnished by the instrument manufacturer and which have the other ends attached to the tripod legs (as is shown in figures 3 and 5) the length of chain required for the adopted spread is 25.27 inches. This corresponds to the length of 128 links of the chain furnished by the manufacturer with the instrument. When this length of chain is used and the movable ring is clamped against the bottom flange of the counterbalance shaft, the constants used in equations (10) and (11) are found to have the following values:

$\alpha = \angle BOD = 13.29°$
$\gamma = \angle AOC = 24.65°$
$r = AC = 12.39$ in.
$w = AO = 24.92$ in.
$q = CO = 29.28$ in.
$s = DO = 27.01$ in.
$a = A'B = A''B = 0.68$ in.
$x_1 = -0.15$ in. (the minus sign meaning that the center of mass of M_1 lies *above* the point of suspension)
$M_1 = 10.01$ lb.
$M_2 = 8.93$ lb.
$L = 5.05$ lb. ft. in. sec.$^{-2}$

Substituting these values of the constants in equations (10) and (11) there results:

$$x_0-0.17\text{ in.}=2.070k\left(\frac{T^2}{H}\right)\frac{z_0-12.39}{z_0}\,\frac{\sin\,(24.65°+\Phi_0)}{\Phi_0\cos\Phi_0}+1.432\times10^{-2}\left(\frac{T^2}{H}\right)\frac{1}{\Phi_0\cos\Phi_0}\tag{10a}$$

where

$$z_0=[1478.4349-1459.3152\cos\,(24.65°+\Phi_0)]^{1/2}$$

and

$$x_0'-0.17\text{ in.}=3.819k'\left(\frac{T^2}{H}\right)\frac{v_0-12.39}{v_0}\,\frac{\sin\,(13.29°+\Phi_0)}{\Phi_0\cos\Phi_0}+1.432\times10^{-2}\left(\frac{T^2}{H}\right)\cdot\frac{1}{\Phi_0\cos\Phi_0}\tag{11a}$$

where

$$v_0=[1463.6445-1346.1748\cos\,(13.29°+\Phi_0)]^{1/2}$$

In order to apply these equations it is further necessary to know where the center of mass of the counterbalance is with reference to, say, the top of the counterbalance, and it is also necessary to know how far some convenient reference point on the theodolite proper is from the point of suspension. The distance of the counterbalance's center of mass from the top of the ring which serves to clamp it to the shaft was found to be 2.53 in. The distance of the counterbalance shaft flange which serves to limit the upward motion of the counterbalance from the point of suspension was found to be 2.22 in. If, therefore, it is desired to use a value of x of, say, 10 inches, the counterbalance would be lowered (10.00−2.22−2.53) in. = 5.25 in. from the uppermost position which it is possible for it to have on the shaft.

To further facilitate the application of these equations to this instrument the table given below has been worked out in the following manner:

An inspection of equations (10a) and (11a) shows that both $x_0+\frac{M_1x_1}{M_2}$ and $x_0'+\frac{M_1x_1}{M_2}$ are directly proportional to $\frac{T^2}{H}$. The factors of proportionality are seen to be composed of two terms. Let the first term of the proportionality factor in equation (10a) be represented by

$$A=k\left[\frac{2.070(z_0-12.39)\sin\,(24.65°+\Phi_0)}{z_0\Phi_0\cos\Phi_0}\right]$$

and the first term of the proportionality factor in equation (11a) be represented by

$$A'=k'\left[\frac{3.819(v_0-12.39)\sin\,(13.29°+\Phi_0)}{v_0\Phi_0\cos\Phi_0}\right].$$

The second terms of the two factors of proportionality are seen to be identical and will be represented by

$$B=\frac{1.432\times10^{-2}}{\Phi_0\cos\Phi_0}$$

Now the proportionality factor to be used in obtaining the required adjustment of the counterbalance is evidently the mean of the two proportionality factors for $x_0+\frac{M_1x_1}{M_2}$ and $x_0'+\frac{M_1x_1}{M_2}$. This mean can, of course, be obtained by adding the mean of the values of A and A' to the value of B. This operation is provided for as follows in the table mentioned:

A set of six spaces has been provided for each whole degree of value of Φ_0 from 1° to 24° inclusive. These six spaces are numbered as is shown in figure 6 and the data intended to be used in these spaces are those indicated in this figure. Hence in order to obtain the required value of the proportionality factor to be used in obtaining $\frac{x_0+x_0'}{2}+\frac{M_1x_1}{M_2}$ for a given angle, the product of the datum given in space 1 and the value of k used is entered in space 4, the product of the datum given in space 2 and the value of k' used is entered in space 5, the mean of the data in spaces 4 and 5 is added to the datum in space 3 and the result of the addition is entered in space 6. It is then only necessary to multiply the datum in space 6 by the proper value of T^2/H and subtract (algebraically) the value of $\frac{M_1x_1}{M_2}$ from this product in order to obtain the required setting of the counterbalance.

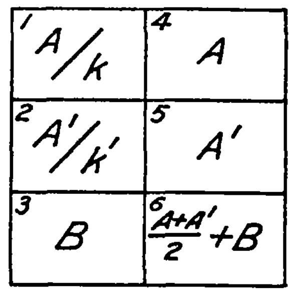

FIGURE 6.

TABLE 1

1°		2°		3°		4°	
1.505 0.959 0.820	----- ----- -----	1.628 1.034 0.410	----- ----- -----	1.672 1.104 0.274	----- ----- -----	1.696 1.176 0.206	----- ----- -----
5°		6°		7°		8°	
1.713 1.244 0.165	----- ----- -----	1.726 1.311 0.137	----- ----- -----	1.736 1.377 0.118	----- ----- -----	1.744 1.440 0.104	----- ----- -----
9°		10°		11°		12°	
1.752 1.503 0.092	----- ----- -----	1.759 1.563 0.083	----- ----- -----	1.765 1.622 0.076	----- ----- -----	1.771 1.679 0.070	----- ----- -----
13°		14°		15°		16°	
1.777 1.734 0.065	----- ----- -----	1.782 1.788 0.060	----- ----- -----	1.788 1.841 0.057	----- ----- -----	1.793 1.893 0.053	----- ----- -----
17°		18°		19°		20°	
1.799 1.943 0.050	----- ----- -----	1.805 1.992 0.048	----- ----- -----	1.810 2.039 0.046	----- ----- -----	1.816 2.086 0.044	----- ----- -----
21°		22°		23°		24°	
1.822 2.132 0.042	----- ----- -----	1.829 2.178 0.040	----- ----- -----	1.836 2.222 0.039	----- ----- -----	1.842 2.266 0.037	----- ----- -----

In computing the quantities given in spaces 1, 2, and 3, the British Absolute system of units has been used, i. e., the unit of mass is the pound and the unit of force is the poundal. Hence, the stiffnesses k and k' are to be determined in poundals per inch, i. e., the stiffness of a spring is found by observing the number of inches of elongation l produced when a mass of m pounds is suspended from it and then multiplying the quantity m/l by the acceleration of gravity, 32.2 ft. sec.$^{-2}$. In determining the value of T^2/H to be used, T is to be measured in seconds and H is to be measured in feet.

Regarding the computation of the proper spring stiffnesses, the guiding principle is to select springs of such stiffness that the counterbalance will be approximately at the mid-point of a convenient adjustment range for the average conditions aboard the ship to be used, and also to select them in such a way that x_0 and x_0' will agree for these prevailing conditions. These computations have been carried out for three types of ships: destroyers, cutters, and battleships or heavy cruisers. For destroyers T^2/H has been assumed to have a value of 1.5 sec^2/ft. and 8° has been chosen as the average value of Φ_0. Selecting $x=15.34$ inches as the midpoint of a convenient counterbalance adjustment range, k should have a value of 5.21 poundals/in. and k' should be 6.30 poundals/in. For cutters T^2/H has been assumed to have 6.0 sec.^{2}/ft. as a mean value and 8° is assumed to be the average amplitude of roll encountered. This, then, gives 1.39 poundals/in. as the value for k and 1.68 poundals/in. as the value for k'. Aboard battleships and heavy cruisers T^2/H has been assumed to have a mean value of 6.0 sec.^{2}/ft. while the mean value of Φ_0 is assumed to be 5°. The required stiffnesses under these circumstances turn out to be 1.38 poundals/in. for k and 1.90 poundals/in. for k'.

An inspection of table 1 will show that for rolling in the neighborhood of 14°, k and k' should have the same value in order to produce good agreement between A and A'. Assuming the same values of T^2/H as before, k and k' should both, therefore, have the value of 5.21 poundals/in. when $10°<\Phi_0<17°$ aboard destroyers. Aboard cutters, their common value will be 1.39 poundals/in. when $10°<\Phi_0<17°$, and aboard battleships and heavy cruisers this common value will be 1.38 poundals/in. when $8°<\Phi_0<17°$. Should the mean amplitude of roll equal or exceed 17°, k' should be given the values 5.21, 1.39, and 1.38 poundals/in. for destroyers, cruisers, and battleships, respectively, while the corresponding values of k will be 5.87, 1.57, and 1.55 poundals/in., respectively.

Except when expressly stated otherwise, it has been assumed in the whole of the foregoing calculations that $\Psi=0$. When this is not the case, the factor $\cos \Phi_0$ appearing in the denominators of the terms on the right hand sides of both equations (10) and (10a) should be replaced by $\cos(\Phi_0-\Psi)$ while this factor in the denominators of the terms on the right hand sides of (11) and (11a) should be replaced by $\cos(\Phi_0+\Psi)$, i. e. assuming that the "athwart ship leg" of the tripod has been placed outboard. To better the agreement between x_0 and x'_0 under these circumstances, the value of k to be used should be taken from the formula

$$k=\frac{1}{(A/k)}\left[\frac{15.17\cos(\Phi_0-\Psi)}{(T^2/H)\cos\Phi_0}-B\right]$$

and the value of k' should be taken from the formula

$$k'=\frac{1}{(A'/k')}\left[\frac{15.17\cos(\Phi_0+\Psi)}{(T^2/H)\cos\Phi_0}-B\right]$$

In these formulae both A/k and A'/k' are regarded as constants, whose value, along with that of B is to be obtained from spaces 1, 2, and 3, respectively, in table 1.

Assuming that the springs having the stiffnesses given by the above formulae have been supplied, spaces (4) and (5) of Table 1 are then to be filled in using these values of k and k'. Next $\frac{H}{T^2}\left(x_0+\frac{M_1x_1}{M_2}\right)$ must be found by adding B, the datum in space (3), to A, the datum in space (4), and then multiplying the result by $\frac{\cos\Phi_0}{\cos(\Phi_0-\Psi)}$. Similarly $\frac{H}{T^2}\left(x_0'+\frac{M_1x_1}{M_2}\right)$ must be found by adding A' to B and multiplying the result by $\frac{\cos\Phi_0}{\cos(\Phi_0+\Psi)}$. The mean of the values $\frac{H}{T^2}\left(x_0+\frac{M_1x_1}{M_2}\right)$ and $\frac{H}{T^2}\left(x_0'+\frac{M_1x_1}{M_2}\right)$ is then, of course, equal to $\frac{H}{T^2}\left(\frac{x_0+x_0'}{2}+\frac{M_1x_1}{M_2}\right)$, the datum to be put in space (6), and the remainder of the procedure is the same as that for the case when $\Psi=0$.

To illustrate the various ways in which table 1 is to be used, the following examples are given:

(1) Suppose that the point of suspension of a theodolite set up on board a battleship lies in the medial plane of the ship and 40 feet above the water plane and that the mean period of roll is 16.2 seconds, the mean amplitude of roll is 6° and that, consequently, the stiffnesses of the springs being used are $k=1.38$ poundals/in. and $k'=1.90$ poundals/in. The entry for space 4 of the 6° section of table 1 is then $1.726\times1.38=2.38$. The entry in space 5 will be the product of 1.311 and 1.90 or 2.49. The mean of these two products is 2.44. Adding this mean to the datum in space 3 we have 2.58 as the entry for space 6 and the completed section of the table for the roll amplitude of 6° will then read

6°	
1.726 1.311 0.137	2.38 2.49 2.58

Hence the required counterbalance adjustment will be

$$\frac{(16.2)^2}{40.0}\times2.58-(-0.17)=17.1 \text{ in.}$$

This means that the counterbalance will be lowered

$$17.1-(2.2+2.5)=17.1-4.7=12.4 \text{ in.}$$

from the topmost position which it is possible for it to have on the shaft (any limitation to the motion of the counterbalance along the shaft furnished by the interference of the tripod legs is not, of course, to be considered in making this adjustment).

(2) Suppose, next, that all conditions are the same as those stated in the first example with the exception that the point of suspension has been moved 30 feet from the medial plane of the ship (the value of Φ being, consequently, $\tan^{-1}\frac{30}{40}=36.9°$ instead of zero as was the case in the first example) and that the spring stiffnesses are, therefore, different from those used in the first example. Assuming that the athwart ship

leg of the tripod (to which the spring with the stiffness k is attached) has been placed outboard and assuming, further, that the same average values of Φ_0 and T^2/H as were used in computing the spring stiffnesses when the point of suspension lay in the ship's medial plane are to be used in computing the spring stiffnesses in this case, we have for the values of these spring stiffnesses:

$$k=\frac{1}{1.713}\frac{15.17\times\cos\,(5-36.9)^\circ}{6.00\cos 5^\circ}-0.165$$
$$=1.16\text{ poundals/in.}$$

and

$$k'=\frac{1}{1.244}\frac{15.17\cos\,(5+36.9)^\circ}{6.00\times 0.9962}-0.165$$
$$=1.39\text{ poundals/in.}$$

The datum entered in space 4 will then be $1.16\times 1.726=2.00$. That to be entered in space 5 will be $1.39\times 1.311=1.82$. For the value of $\frac{H}{T^2}\left(x_0+\frac{M_1x_1}{M_2}\right)$ we have:

$$(2.00+0.137)\frac{\cos 6^\circ}{\cos\,(6^\circ-36.9^\circ)}=2.48$$

and for the value of $\frac{H}{T^2}\left(x_0+\frac{M_1x_1}{M_2}\right)$ we have:

$$(1.82+0.137)\frac{\cos 6^\circ}{\cos\,(6^\circ+36.9^\circ)}=2.66$$

Hence for the datum to be entered in space 6 we take the mean of 2.48 and 2.66 which is 2.57. The completed section for the 6° amplitude of roll then reads:

6°	
1.726 1.311 0.137	2.00 1.82 2.57

Multiplying the datum in space 6 by T^2/H and subtracting the value of $\frac{M_1x_1}{M_2}$, we have

$$\frac{2.57\,(16.2)^2\cos 36.9^\circ}{40}-(-0.17)=13.7\text{ in.}$$

as the required counterbalance adjustment. As before, this means that the counterbalance must be lowered $13.7-4.7=9.0$ in. from its topmost position on the shaft.

In figure 3 it will be noticed that instead of having only a spring stretching between A and C, a spring with a short piece of chain attached to it is stretched between these two points. If the spring is fastened to the counterbalance shaft, and the chain is fastened to the tripod leg, this arrangement will eliminate any torques that might arise on account of the buckling of the springs. Furthermore, the part of the distance covered by the length of the spring should be very nearly the same as that shown in the figure. This is true because a spring whose length is much greater than that shown will buckle for large swings of the theodolite, while the elastic limit of a spring which is very much shorter than the one shown is more apt to be exceeded for large swings of the theodolite than is the case for the longer spring. Hence if 10 links of the kind of chain furnished by the manufacturer for limiting the spread of the tripod legs is used with a spring 9.22 in. long (9.22 in. is the distance between the inside edges of the spring couplings when the spring is unstretched), the combined length of the spring and chain assembly (including the length of the two hooks which are linked with the eyelets at A and C) will be equal to 12.39 in.—which is the length of AC required.

In order to secure springs of the proper length and stiffness it will, in general, be necessary to make use of the formula giving the stiffness of a closed coil helical spring. This formula is

$$k=\frac{d^4G}{64np^3}$$

where d is the diameter in inches of the wire used, G is the coefficient of rigidity in poundals per square inch, n is the number of turns used and p is the mean radius of the coil in inches. If phosphor-bronze (the alloy usually used in making such springs) is used, G has the value 1.996×10^7 poundals per square inch.

Finally, the effects on the motion of the theodolite of variations of T and Φ_0 from the assumed mean values may be determined. In order to do this it is necessary to make use of the more general torque equations, i. e., those equations which allow for other values of θ than that of zero. These equations are:

$$kqw\frac{z-r}{z}\sin(\gamma+\Phi_0+\theta)+Mgh\sin\theta+L$$
$$=Mh\Phi_0H\frac{4\pi^2}{T^2}\cos\,(\Psi+\Phi_0+\theta)\qquad(14)$$

where

$$z=[q^2+w^2-2qw\cos(\gamma+\Phi_0+\theta)]^{1/2}$$

and

$$2k'sw\frac{(v-r)}{v}\sin(\alpha+\Phi_0+\theta)+Mgh\sin\theta+L$$
$$=Mh\Phi H\frac{4\pi^2}{T^2}\cos\,(\Psi+\Phi_0+\theta)\qquad(15)$$

where

$$v=[a^2-2a\sqrt{q^2-s^2}+q^2+w^2-2ws\cos(\alpha+\Phi_0+\theta)]$$

It is at once evident that it would be very difficult to solve these equations for θ_0 using T and Φ_0 as the independent variables. θ and Φ_0 can, however, be used as independent variables and the corresponding values of T ascertained. Using a mean period of $T=15.58$ sec. and a value of 5° for the mean value of Φ_0, the values of T which will give $-3°$, $-2°$, $-1°$, 0°, 1°, 2°, and 3° as values of θ have been computed for each of the cases in which Φ_0 takes on the values of 5°, 9°, 13°, 17°, and 19°—this being done both for the case where the single spring is being stretched and also for the case where the two springs are being stretched, and the resulting values of T being designated as T' and T'' respectively.[5] Using the same assumed period (15.58 sec.) but a value of 9° for the mean value of Φ_0, the values of T which will cause θ to vanish when $\Phi_0=5°$, 9°, 13°, 17°, and 19° have also been computed. This latter set of computations has also been carried out for the case in which the counterbalance has been adjusted for 17° as a mean value of Φ_0. Also the values of T, which will give $-3°$, 0°, and $+3°$ as values of θ when mean values of T and Φ_0 of 15.58 sec. and 19°, respectively, are assumed, have been computed for the cases in which Φ_0 takes on the values 5°, 9°, 13°, 17°, and 19°. Finally, assuming a mean value of $T=11.5$ sec. and 5° as a mean value of Φ_0, the values of T which will give $-3°$ and $+3°$ as values of θ have been computed for the 5°, 9°, 13°, 17°,

[5] Ψ was arbitrarily given the value zero in these computations, as well as in all the computations succeeding them.

and 19° values of Φ_0. The results of all of these computations are shown in the following tables:

TABLE 2

Assumed mean values: $\Phi_0=5°$, $T=15.58$ sec.

	Φ_0	5°	9°	13°	17°	19°
$\theta=+3°$	T'	7.28	8.98	10.02	10.71	10.97
	T''	7.12	8.44	9.06	9.32	9.38
$\theta=+2°$	T'	8.47	10.20	11.19	11.81	12.04
	T''	8.32	9.58	10.03	10.16	10.17
$\theta=+1°$	T'	10.53	12.09	12.87	13.31	13.46
	T''	10.40	11.30	11.41	11.27	11.16
$\theta=0°$	T'	15.58	15.64	15.61	15.56	15.53
	T''	15.58	14.90	13.51	12.74	12.51
$\theta=-1°$	T'	--------	27.20	21.41	19.49	19.41
	T''	--------	23.38	17.40	15.16	14.45
$\theta=-2°$	T'	--------	--------	59.51	29.51	26.25
	T''	--------	--------	29.34	19.50	17.64
$\theta=-3°$	T'	--------	--------	--------	--------	89.51
	T''	--------	--------	--------	32.64	24.52

TABLE 3

Assumed mean values: $\Phi_0=9°$, $T=15.58$ sec.

	Φ_0	5°	9°	13°	17°	19°
$\theta=0°$	T'	14.98	15.24	15.10	14.97	14.93
	T''	17.37	16.18	15.19	14.68	14.08

TABLE 4

Assumes mean values: $\Phi_0=17°$, $T=15.58$ sec.

	Φ_0	5°	9°	13°	17°	19°
$\theta=0°$	T'	15.48	15.51	15.48	15.42	15.39
	T''	18.94	17.66	16.57	15.72	15.37

TABLE 5

Assumed mean values: $\Phi_0=19°$, $T=15.58$ sec.

	Φ_0	5°	9c	13°	17°	19°
$\theta=+3°$	T'	7.28	9.00	10.04	10.73	10.99
	T''	7.63	9.31	10.19	10.67	10.81
$\theta=0°$	T'	15.68	15.71	15.67	15.62	15.59
	T''	19.23	17.88	16.79	15.93	15.56
$\theta=-3°$	T'	--------	--------	--------	--------	101.05
	T''	--------	--------	--------	--------	--------

TABLE 6

Assumed mean values: $\Phi_0=5°$, $T=11.5$ sec.

	Φ_0	5°	9°	13°	17°	19°
$\theta=+3°$	T'	6.41	7.68	8.39	8.84	9.00
	T''	6.21	7.08	7.39	7.47	7.53
$\theta=-3°$	T'	--------	--------	31.61	19.84	18.20
	T''	--------	--------	21.08	14.42	13.08

5° and 19° were selected as mean values of Φ_0 in these computations since, with the use of the method of selecting spring stiffnesses previously outlined, x_0 and x'_0 agree for these mean values of Φ_0. Similarly 9° and 17° were also selected as mean values of Φ_0 due to the fact that x_0 and x'_0 differ most for these mean values of Φ_0 when the spring stiffnesses just mentioned are used. $-3°$ and $+3°$ were selected as limiting values of θ because the theodolite for which these computations apply has a field of view of 6° and any greater values of θ (taken absolutely) would, of course, carry the balloon out of this field.

It will be noted that no value of T' or T'' is given for a good many cases where θ was assumed to have a negative value. If, for instance, equation (14) is solved for T^2, the expression representing its value will be a fraction having the coefficient of $\frac{1}{T^2}$ in equation (14) as the numerator and the left-hand side of (14) will constitute the denominator. Whenever the denominator of this fraction took on a negative value the omissions just referred to were made. This was done because the values of T in these cases became imaginary unless cos $(\Psi+\Phi+\theta)$ became negative in the numerator. Since, if for no other reason, the mechanical arrangement of the theodolite would prevent this, it was not thought worth while to finish the computations for these negative denominator values.

An inspection of these tables shows that for $\theta=0°$, T' in contrast to T'', remains practically constant throughout the whole of the range of the values of Φ_0 chosen—this being true regardless of the assumed mean value of Φ_0. It will further be seen that, as was to be expected, these values of T' differ more from the assumed values of T for $\Phi_0=9°$ and $\Phi_0=17°$ than is the case for the assumed values $\Phi_0=5°$ and $\Phi_0=19°$—the error being due, of course, to the fact that the counterbalance setting corresponds to the mean of the values of x_0 and x'_0 and to the fact that these two quantities differ most when the assumed mean values of Φ_0 are 9° and 17° respectively. In view of these two facts then, it is apparent that the substitution of a second spring lying in the athwartship plane of the ship for the two springs not lying in that plane would greatly improve the results secured. A very simple way to construct a spring and chain assembly of this kind is to fasten the chain end to a ring or toroid which can in turn be fastened to the tripod by resting it on the tripod legs and providing stops to prevent its sliding up the legs. Since considerable work is involved in preparing tables similar to table 1, it will, of course, be advantageous to design this ring and to locate the stops in such a way that the triangles formed by the point of suspension and the pivoting points of the spring and chain assembly are congruent to the corresponding triangle shown in figure 3. Should it be desired to use a device of this kind, the necessary trigonometrical calculations for its design have been carried out and may be had upon request.

Furthermore if such a two-spring assembly is used instead of the usual three-spring assembly, tables 2 and 5 in conjunction with the roll record of the U. S. S. *Pensacola* previously referred to indicates the possibility of a considerable reduction in the optical field of the theodolite together with the consequent possibility of an increase in the theodolite's magnification. To see this, reference need only be made to figure 7. In this figure the curves marked $\theta=1°$, 2°, 3°, $-1°$ and $-2°$ indicate the amplitudes and periods necessary to produce the designated values of θ when (1) the two-spring assembly described above is used, (2) T is assumed to have a mean value of 11.5 sec., and (3) the mean value of Φ_0 is assumed to be 3°. The scattered points in the figure indicate the values of the various amplitudes and periods of roll observed aboard the *Pensacola* in the set of observations previously referred to.[3] When it is now considered that the natural period of roll of the *Pensacola* is 11.5 sec., it will be seen that a fact which is known to those familiar with the behavior of ships at sea is well illustrated here, i. e., that, excluding storm conditions, virtually all heavy rolling has a period which is closely coincident with the natural period of roll. If, therefore, the natural period of roll is used in obtaining the counterbalance adjustment from table 1 instead of the average period as obtained from timing a comparatively small number of rolls of the ship (which may, of course, differ considerably from the natural period), the large amplitudes of roll of the ship will have an increase in magnification would only mean that the field of view of the shipboard theodolite would have to be reduced from 6° to 2.4°.

Summing up then, under the assumptions made it has first been shown that, due to the scattering of the periods of roll depicted in figure 7, the position of the counterbalance with the theodolite swinging free is immaterial. Secondly, under somewhat more general assumptions it has further been shown that it is possible to obtain a fairly satisfactory amount of stability for the theodolite by connecting the end of the counterbalance shaft to the tripod legs with light springs and coordinating the adjustment of the counterbalance with the stiffnesses of the springs

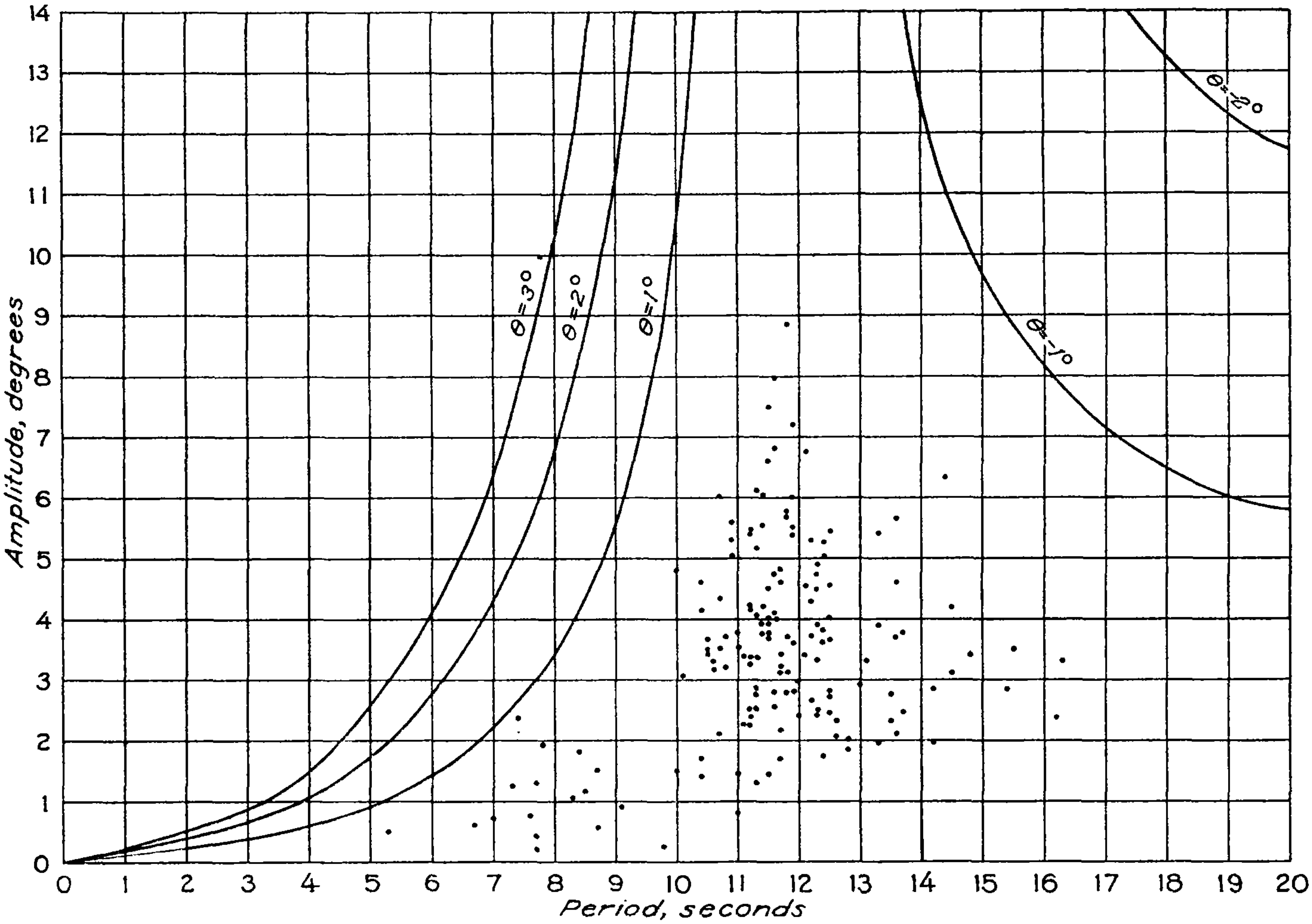

FIGURE 7.

very little effect on the verticality of the theodolite.[4] When this fact is considered along with the fact that of the 155 amplitudes and periods of roll observed, not a single roll was sufficient to produce a 1° deviation of the theodolite from the vertical under the circumstances described above, the possibilities as to the reduction of the field of view of the theodolite along with the consequent increase in magnification are evident. It seems quite possible, in fact, that the 20 power magnification possessed by the ordinary "shore" theodolite may, under these circumstances, be available for use in the shipboard theodolite since such used. Finally, using the same set of more general assumptions, it has also been shown that a much more satisfactory amount of stability can be secured by using two springs instead of three to counteract the inertial torque on the theodolite and that it seems probable that this stability will be sufficiently great to warrant a considerable decrease in the theodolite's field of view along with a corresponding increase in the magnification secured.

[3] These data are published with the kind permission of the Chief of the Bureau of Construction and Repair of the Navy Department.

[4] It is evident that, under these circumstances, the obtaining of the proper counterbalance adjustment is greatly simplified. For T being taken as a constant for a given ship and H being taken as a constant for any one position of the theodolite on the ship, the counterbalance adjustments themselves may be entered in space 6 of the amplitude of roll sections of table 1 and, hence, a simple reference to the table thus filled in will replace the computations previously indicated.

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PRELIMINARY REPORTS ON TORNADOES IN THE UNITED STATES DURING 1938

By J. P. Kohler

[Weather Bureau, Washington, February 2, 1939]

The present study is based largely on the data contained in table 3, severe local storms, appearing in the several issues of the Monthly Weather Review published during the year 1938. A final and more detailed study will be published in the *United States Meteorological Yearbook, 1938*. The figures given here are substantially correct; however, it must be remembered that all are subject to change after the final study mentioned above.

In contrast to the previous year, 1938 brought some highly destructive tornadoes; none of them, however, compared in violence or severity with the Georgia tornadoes of 1936, or the outstanding St. Louis storm of 1927 or the tri-State tornado of 1925. Table 1 shows that 189 tornadoes occurred in 25 States; during the preceding year the total number reported was 123, a difference of 66 storms. The number of deaths caused by tornadoes in 1938 was reported as 178; this is a considerable excess over the comparatively small figure of 29, which was the tornado death toll in 1937. Injuries suffered from tornadoes in 1938, based on all available reports, were in excess of 2,189; for the preceding year the list of injured was only 192. Total property damage resulting from the 1938 tornadoes is estimated at nearly $8,000,000.

Table 1, herewith, enumerates tornado frequency, deaths, injuries, and damage figures, by States during the year. It will be seen that the greatest number of tornadoes occurred in March with a total of 60 storms which is 44 greater than the 22-year average (1916–37) for that month. May was second in tornado frequency with 42 storms, an excess of 11 above normal; in April there were 26 or 3 more than normal. Thirteen tornadoes occurred in July, 8 each in August, September, and November, 2 in February, and 1 in December. October was the only month in which no storm of even possible tornadic character was reported.

The greatest loss of life resulting from tornadoes during the year occurred in March when 74 deaths were reported. There were 32 deaths in September, 21 in February, 17 in April, 15 in May, 14 in June, 3 in July, and 2 in November. The tornadic activity of August and December caused no loss of life.

Practically all of the total property loss, approximately $7,790,000, occurred during the months of March, June, and September. The greatest monthly loss was in September and was due primarily to the series of destructive tornadoes which occurred in Charleston, S. C., and vicinity on the 29th.

Some of the most outstanding tornadoes (from the point of view of loss of life and property damage) in the United States during the year are as follows: In Kansas on March 10 a single tornadic disturbance caused 10 deaths, 150 injuries, and damage of about $575,000. In Arkansas in the same month 1 tornado on the first, 4 on the 28th and 9 on the 30th resulted in 18 deaths, 287 injuries and damage of $366,000. In Missouri during March, 4 tornadoes on the 15th caused 11 deaths, 27 injuries and damage estimated at $257,000; and near the close of the month, on the 29–30th, 8 tornadoes were responsible for 5 deaths. 56 injuries and property damage exceeding $224,000. On March 15 a series of 3 destructive tornadoes in Illinois caused 10 deaths, 77 injuries, and property damage to the extent of $215,000, and 3 storms on the 30th resulted in 13 deaths, 89 injuries, and damage of more than $1,500,000. On the same date a series of 6 minor tornadoes caused 4 injuries, but no deaths, and property damage estimated at $59,000.

The most outstanding instance of destructive tornadic action during the year occurred in Charleston, S. C., and vicinity on September 29 when a series of 5 tornadoes in Charleston County killed 32 persons, injured 150 or more, and damaged property to the estimated extent of $2,000,000. In this connection it may be stated that this is the greatest loss of life and property caused by tornadoes within a restricted area in that State during the period for which records are available.

In the event that the possible tornadoes enumerated in Table 2 are later adjudged to be true tornadoes, the 1938 figures will be 200 tornadoes, 181 deaths, 1,321 injuries, and property losses exceeding $8,045,000. (See tables 1 and 2 on pp. 413–414.)